

Can Universe Experience Many Cycles with Different Vacua ?

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Abstract

Recently, the notion that the number of vacua is enormous has received increased attentions, which may be regarded as a possible anthropical explanation to incredible small cosmological constant. Further, a dynamical mechanisms to implement this possibility is required. We show in an operable model of cyclic universe that the universe can experience many cycles with different vacua, which is a generic behavior independent of the details of the model. This might provide a distinct dynamical approach to an anthropically favorable vacuum.

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Recently, the notion that the number of vacua is astronomical has received increased attentions, which may attribute to two developments of observations and theory. The first is the accelerated expansion of present universe implied by supernovae SNIa [1], to which the simplest explanation is a small cosmological constant. However, its incredible smallness and fine tuning make us hard to find a vacuum in a range of observed value unless there are an enormous of solutions with almost all possible values. The second is that it is known that there are a large number of vacuum states in string theory [2, 3], see also Ref. [4, 5]. Recently the space of all such vacua has been dubbed landscape [6]. In some sense, the low energy properties of string theory can be approximated by field theory. Thus the landscape can also be described as the space of a set of fields with a complicated and rugged potential, where the local minima of potential are called the vacua. When this local minimum is an absolute minimum, the vacuum is stable, and otherwise it is meta stable. The value of potential energy at the minimum may be regarded as the cosmological constant for that vacuum. Since the number of vacua is exponentially large, there can be enough vacua with small cosmological constant. The probability that an observer find a cosmological constant no greater than the value observed will be not too small, which is often referred as an interesting application for anthropic principle [7, 8], see Ref. [9] for a recent comment.

Though string theory brings us diverse vacua, which makes us possible to solve the problem of cosmological constant puzzling us for a long time, we still require a dynamical mechanism to implement this possibility. Eternal inflation [10] based on inflation scenario [11, 12] is often regarded as a natural candidate, where an observer starts with a large value of vacuum, with bubble nucleating over potential barrier he will see a series of vacuum descending. The chances that he lands in an observable vacuum may be small, but still acceptable when we consider possible infinite numbers of bubbles. But the realistic evolution of universe controlled by a landscape might show itself more complex than that expected, which may spur our more thinks. Note that recently, the cyclic model has been proposed as a radical alternative to inflation scenario [13], which is motivated by the string/M theory, where the relevant dynamics can be described by an effective field theory. The separation of the branes in the extra dimensions is modeled as a scalar field. The idea that universe is cyclic is ancient, which can be found in mythologies and philosophies dating back to the beginning of recorded history of mankind, and has been still an aesthetic attraction. Thus with diverse environments which string theory is likely to bring us, a naive question

is whether the universe can experience many cycles with different vacua.

In string landscape there are a large number of dS minima as well as AdS minima. The cosmology of scalar field with AdS minimum has been investigated in Ref. [14] extensively. It is well known that for an expanding universe with dS minimum, it will arrive at a dS regime asymptotically. However, the universe with AdS minimum can not approach an AdS regime, and instead it will stop expanding and enter into a collapsing phase. During the collapsing process, its energy goes up rapidly. The singularity will become inevitable unless there is a mechanism responsible for the bounce. It has been noticed [15] that when the universe is in the contracting phase, the field can be driven and roll up along its potential, and arrive at a large enough value for a successful inflation to occur in expanding phase [16]. These pioneer attempts have given us a partial answer. In the following we will focus on a simple example that allows us to obtain our basic conclusion. We show that the universe can experience many cycles with different vacua and further explain how a cycle with an early successful inflation and a small cosmological constant may be anthropically selected as the universe in which we live.

Let us start with such an effective Lagrangian of single scalar field as follows

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \Lambda_* \left(1 - \cos\left(\frac{m}{\sqrt{\Lambda_*}}\varphi\right)\right) + \Lambda \quad (1)$$

where Λ is a small positive constant which makes the minima of periodic potential AdS's. The potential is plotted in the left-up of Fig. 1.

No losing generality, we take the field around the origine as example, where the potential can be written as $\frac{1}{2}m^2\varphi^2 - \Lambda$. We assume that initially the universe is in a contracting phase and the field φ is in $\varphi \simeq 0$ minima of its potential and has a small $\dot{\varphi}$ responsible for $\rho_\varphi > 0$. In this case $m^2 \gg h^2 \equiv \left(\frac{\dot{\varphi}}{a}\right)^2 \sim \rho_\varphi$, where $8\pi/m_p^2 = 1$ has been set, the term $3h\dot{\varphi}$ can be neglected. Thus the equation of motion of φ can be reduced to

$$\ddot{\varphi} + m^2\varphi \simeq 0 \quad (2)$$

which can be solved as an oscillation with frequency m , $\varphi \simeq \varphi_a \sin(mt)$, where φ_a is the amplitude of the oscillation. When taking the time average over many oscillations of field, $\langle p_\varphi \rangle \simeq 0$ is obtained. Thus around the minimum of the field the behavior of the universe is similar to the case dominated by usual matter.

In a collapsing universe, $3h\dot{\varphi}$ is anti-frictional. Instead of damping the motion of φ in the expansion, it accelerates the motion of φ and makes the energy of field increase.

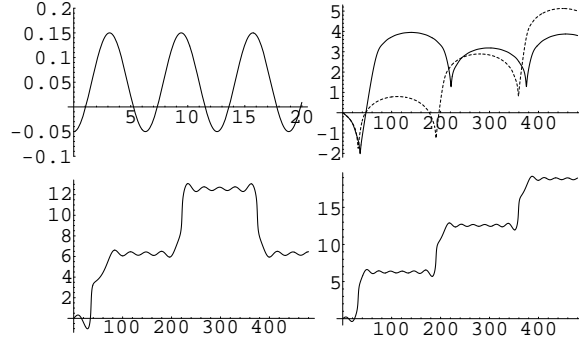


FIG. 1: The upper left panel is the figure of potential with $\Lambda_* = 0.1$, $\frac{m}{\sqrt{\Lambda_*}} = 1$ and $\Lambda = 0.05$. The upper right panel is the figure of $\ln a$ with respect to time. The dashed and solid line are the cases that the hight Λ_* of potential are taken as 0.07, 0.04, respectively, where $\Lambda = 0.005$ is taken. The lower panel are the figures of field with respect to time, where $\sigma = 1$ is taken. We find that for different cycles of universe the field oscillates in different minima.

Since $h \simeq m\varphi_a$ and $\dot{\varphi} \simeq m\varphi$, when $\varphi \gtrsim 1$, we have $3h\dot{\varphi} \gtrsim m^2\varphi$. Thus with the increase of amplitude of oscillating field, when the term $3h\dot{\varphi}$ can not be neglected any more, the oscillation of field ends and the universe will enter into the regime dominated by kinetic energy of field rapidly, $\dot{\varphi}^2 \gg m^2\varphi^2$, *i.e.* the detail of potential has been not important, thus $p_\varphi \simeq \rho_\varphi$. Instead of Eq. (2),

$$\ddot{\varphi} + 3h\dot{\varphi} \simeq 0. \quad (3)$$

can be satisfied. This leads to $\rho_\varphi \sim \dot{\varphi}^2 \sim 1/a^6$, thus we obtain

$$a^3 \simeq c\sqrt{\frac{3}{2}}(t_s - t) \quad (4)$$

where c is an integral constant and t_s is the time when $a \simeq 0$, which means that the universe will collapse into singularity unless the bounce occurs, and

$$\varphi \simeq \varphi_k + \sqrt{\frac{2}{3}} \ln\left(\frac{t_s - t_k}{t_s - t}\right) \quad (5)$$

where φ_k and t_k are the values of field and time just entering into the regime dominated by kinetic energy, respectively.

We expect that Big Crunch singularity is not a possible feature of quantum gravity and there might be some mechanism from high energy/dimension theories responsible for a non-singular bounce. We suppose, following this line, that in high energy regime the Friedmann

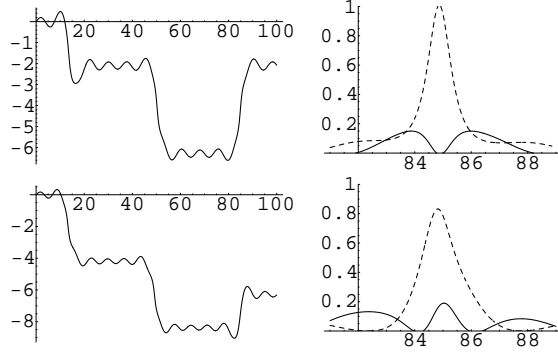


FIG. 2: The left side panels are the figures of field with respect to time, while the right side panels are the figures of its potential energy (solid line) and kinetic energy (dashing line) in time interval (81, 89). The upper right panel is the case that the field strides over two hills and one valley and the lower right panel is the case of one hill and one valley.

equation can be modified as $3h^2 \simeq \rho_\phi - \rho_\phi^2/\sigma$ [17], which is only required to carry out visual numerical calculations. The final results are independent on this detail that the bounce is implemented. During the contraction, the field will be driven up along its potential. When it arrives at apex of potential hill, it will roll down to another minimum. Due to the endless growing of its kinetic energy, in some time the modification term of Friedmann equation will become important, and when $\rho \simeq \sigma$, we obtain $h \simeq 0$, thus the collapse of universe will be hold back and then enter into an expanding phase. The behavior of scale factor around the bounce can be solved approximately as $a^6 \simeq 3c^2(t - t_b)^2/2 + c^2/2\sigma$ where t_b is an integral constant. When $t \simeq t_b$, the scale factor of universe arrives its minimum.

The universe, controlled by multi AdS minima and a bounce mechanism in high energy, will show itself many oscillations. We plot the functions of the field and scale factor with respect to time in Fig. 1, where when the modification scale σ is large than the height of potential hill, the field will be driven from a minimum to another, and in each cycle of oscillating universe, the field will generally lie in different minima [18].

The value to which the field is driven during each contraction/expansion cycle can be simply estimated. When the field just enters into the regime dominated by kinetic energy, $\dot{\varphi}_k^2 \simeq m^2\varphi_k^2$ can be satisfied approximately, and when the field is about at the point of the

bounce, its value is $\dot{\varphi}_b^2 \simeq \sigma$. Thus from Eq. (4),

$$\left(\frac{t_s - t_b}{t_s - t_k}\right)^2 = \left(\frac{a_b}{a_k}\right)^6 = \left(\frac{\dot{\varphi}_k}{\dot{\varphi}_b}\right)^2 \simeq \frac{m^2 \varphi_k^2}{\sigma} \quad (6)$$

can be obtained. When it is substituted into Eq. (5), the maximum value is given by

$$\varphi \simeq \varphi_k + \sqrt{\frac{2}{3}} \ln\left(\frac{\sigma}{m^2 \varphi_k^2}\right) \quad (7)$$

where the second term has been doubled since during kinetic dominated period the behavior of the field can be regarded approximately as symmetrical before and after the bounce, which can also be seen from figures of right side of Fig. 2. In general $\varphi_k \simeq 1$, thus according to Eq. (7), the change of field can be written as

$$1 + \frac{1}{2} \sqrt{\frac{2}{3}} \ln\left(\frac{\sigma}{m^2}\right) \lesssim \Delta\varphi \lesssim 1 + \sqrt{\frac{2}{3}} \ln\left(\frac{\sigma}{m^2}\right) \quad (8)$$

which is only determined by the mass m around its minima and the modification scale σ . In Fig. 1, $\sigma = 1$ and $m^2 = 0.07, 0.04$. From Eq. (8), $2.5 < \Delta\varphi < 3.5$ is obtained, which makes the field just stride over a hill during each cycle and can be seen from Fig. 1, where the distance between the apex of hill and the valley is 3.14.

Further, these results can be straightly applied to the case with multi degrees of scalar freedom, whose potential in field space is reflected in a complicated terrain in real space. In Fig. 3, the potential is plotted as a function of two scalar fields, where there are many hills and valleys with AdS and dS minima [19]. The similar behaviors can be found. Initially the fields are in some valley with AdS minimum of landscape and oscillate [20]. When the energy of oscillation can be compared with this minimum, the universe will stop expanding and enter into a collapsing phase. During the contraction, the kinetic energy of the fields will rise rapidly and be far larger than their potential energy, which can be seen from the dashed line of Fig. 3. When the total energy arrives the modification scale, $h \simeq 0$, the universe will return and expand. With the expansion of universe, the kinetic energy of fields decreases rapidly. Finally the fields will be “dropped” into some hill or valley, which is generically different from previous one. This scenario can be visualised as that an airplane takes off from a valley, and as close to cloud of modification scale, it returns and lands on some hill [21] or in some other valley, see Fig. 3. During this period the number of hills and valleys it flies over is determined by the detail of the landscape and the bounce scale. For the case that the landscape has more AdS minima than dS’s, or even if the field can stay in a positive

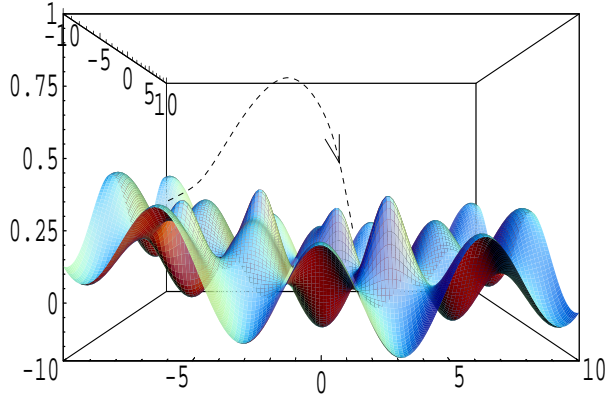


FIG. 3: The 2D landscape. The potential is plotted as a function of two scalar fields. The dashed line is the evolution of total kinetic energy of fields during some cycle.

potential temporarily it will also eventually roll to minimum with negative potential (for further illustration see Fig. 4), periodical fly/landing process will be inevitable and highly effective. In each cycle, the universe lies generically in different states.

The quantum and thermal fluctuations will increase during the contraction of each cycle. However, the increase of their energy is less fast than that of the field energy $\sim 1/a^6$. Thus as long as initially the fluctuations energy is lower than the background energy, it will not exceed forever. The form of black holes may be a problem [23], and further black hole gas accumulated [24] after some cycles may make the universe cease to cycle. However, there is a possibility that the field could land on some potential hill or nearly flat plain after one or several cycles. In this case there is a large room for inflation to occur, which may dilute these leftovers not required and seed primordial perturbations responsible for structures of observable universe. Since the landscape is a complex function with many scalar fields (degrees of freedom), from a hill to a bottom of a valley, a large number of paths can be expected to implement inflation. It has been pointed out [25] that in this case inflation can be driven by several distinct fields, but the rolling of only one field is dominant in each segment of path. When the minimum of valley which the field rolls to finally is AdS's, the universe will collapse eventually and begin to the next cycle. Such a cycle (fly, landing then (not) inflation) can occur time after time. When the number of cycles is large or approaches infinity, whichever minimum initially the universe is in, it can run over almost all vacua of the landscape. Thus for a given landscape if there are some vacua suitable to us, there are

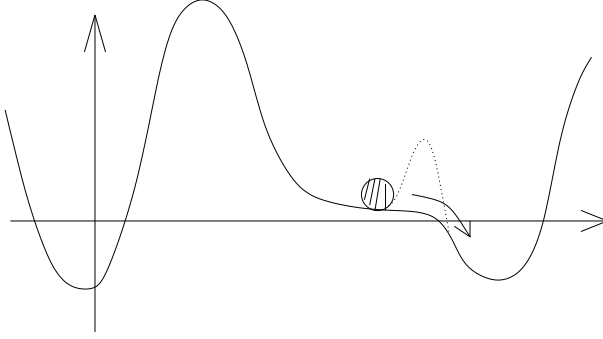


FIG. 4: When the minima of valleys which the field rolls to finally are dS minima, the cycle will generally cease, (or occur in subsequently nucleated bubbles). To make universe cycle more effectively, the potential (solid line) in the figure is a feasible example. In almost all cases the field enters into negative potential not by tunnelling over hill (dashed line) but by rolling, which is locally just the case in cyclic model [22]. But different from it, here the universe will be in different regions of potential of fields in each cycle. Not in all cycles are there such local potentials making field show a quintessence-like behavior at present as in cyclic model [13]. However, when the number of cycles is large or approaches infinity, some cycles similar to the universe which we live in will be possible.

certainly some cycles where we can live. In some extent our universe with an early successful inflation and a late time small cosmological constant may be regarded as an anthropic one of many cycles.

In conclusion, we show that the universe can experience many cycles with different vacua, which may be regarded as a feasible approach to an anthropically favorable vacuum. In our example the universe is controlled by a periodic scalar field potential with AdS minima. We find that during each contraction/expansion cycle of universe, the field will be driven from a minimum to another. The usual anthropic proposal for small observed cosmological constant assumes that there are many universes, or in the universe there are many different regions based on eternal inflation [26, 27], while in our proposal, the universe experiences many cycles and in each cycle it lies in different vacua. It is in this sense that it might be another interesting dynamical implement different from eternal inflation to solve the problem of cosmological constant. We in this paper only simply used a known modification of Friedmann equation to avoid Big Crunch. Though our study might be idealistic, we

believe that we have identified the basic ingredient of the required answer. A more realistic bounce mechanism expected from possible quantum gravity theory is required, which is also significant to avoid a potentially catastrophic instability of the landscape with meta-stable dS minima [21]. Some further issues, especially how to embed it into string/M theory, and interesting applications is left in future works.

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